1. Thermodynamic and Fluid-Dynamic Concepts

Working Fluid

A working fluid of gas turbine engine is a pressurized gas that actuates an engine. Examples include an air and combustion gases.

More generally, in a thermodynamic system, the working fluid is a liquid or gas that absorbs or transmits energy.

The flow of the working fluid is characterized by values of mass flow rate G through the considered cross-section of the flowpath and area F of this cross-section:

G - mass flow rate, $\frac{\mathrm{kg}}{\mathrm{s}}$;

F - area of a flowpath cross-section, m^2.

Gas State Properties

The working fluid properties are essential for the full description of thermodynamic systems. Although working fluids have a very large number of physical properties which can be defined, but there are few thermodynamic properties which are required in thermodynamic design and analysis. Pressure, temperature, entropy, density and specific volume are most common.

p - absolute pressure, $\mathrm{Pa} = \frac{\mathrm{N}}{\mathrm{m}^2}$;

T -absolute temperature, K;

ρ - density, $\frac{\mathrm{kg}}{\mathrm{m}^3}$;

ν - specific volume, $\frac{\mathrm{m}^3}{\mathrm{kg}}$;

s - entropy, $\frac{\mathrm{J}}{\mathrm{kg}\cdot\mathrm{K}}$.

Traditionally the entropy has been shrouded in mystery, primarily due to being less concrete than the other properties. During compression or expansion the increase in entropy is a measure of the thermal energy loss, which becomes unavailable as useful work.

Physical properties of gas

μ - Molecular Weight, $\frac{\text{kg}}{\text{mol}}$.

Molecular weight for a pure gas is defined in the Periodic Table. For mixtures of gases, such as air, the molecular weight may be found by averaging of components on a molar basis.

A mole is the quantity of a substance such that the mass is equal to the molecular weight in grammes. For any perfect gas one mole occupies a volume of 22.4 liters at 0°C and 101.325 kPa.

A mole contains the Avogadro's number of molecules ($6.023 \cdot 10^{23}$).

c_p - Specific Heat Capacity at Constant Pressure, $\frac{\mathrm{J}}{\mathrm{kg}\cdot\mathrm{K}}$.

c_v - Specific Heat Capacity at Constant Volume, $\frac{\mathrm{J}}{\mathrm{kg}\cdot\mathrm{K}}$.

These are the amounts of energy required to raise the temperature of one kilogram of the gas by 1°C, at constant pressure and volume respectively.

For gas turbine engines with a steady flow of gas (as opposed to piston engines where it is intermittent) the specific heat capacity at constant pressure c_p is usually used.

Specific heat capacity of ideal gases is a function of gas composition and static temperature.

R - Gas Constant, $\frac{\mathrm{J}}{\mathrm{kg}\cdot\mathrm{K}}$.

The gas is equal to the difference between c_p and c_ν:

$$c_p - c_\nu = R .$$

The gas constant for an individual gas is the universal gas constant divided by the molecular weight

$$R = \frac{R_\mu}{\mu} .$$

The universal gas constant R_μ has a value of $8.3143 \frac{\mathrm{J}}{\mathrm{mol}\cdot\mathrm{K}}$.

k - Isentropic Ratio.

Isentropic ratio is the ratio of the specific heat capacity at constant pressure to that at constant volume:

$$k = \frac{c_p}{c_v}.$$

It is also a function of gas composition and static temperature.

In approximate calculations, we can take the following values of the properties of the working fluid:

- air

$$R_a = 287.0 \frac{\mathrm{J}}{\mathrm{kg \cdot K}}, \qquad c_{pa} = 1005 \frac{\mathrm{J}}{\mathrm{kg \cdot K}}, \qquad k_a = 1.4;$$

- combustion products of aviation kerosene

$$R_{cg} = 287.5 \frac{\mathrm{J}}{\mathrm{kg \cdot K}}, \qquad c_{pcg} = 1159 \frac{\mathrm{J}}{\mathrm{kg \cdot K}}, \qquad k_{cg} = 1.33.$$

Equation of State

The ideal gas equation of state is:

$$p = \rho \cdot R \cdot T$$

or

$$p \cdot \nu = R \cdot T \ .$$

The study of gases has resulted in certain laws. In discussing these laws of behavior, gases are referred to as being either ideal or real. The ideal gas is only hypothetical and obeys various simplified laws that the real gas can only approach under certain conditions.

All gases employed as the working fluid in gas turbine engines, except for water vapor, may be considered as perfect gases without compromising calculation accuracy. When the mass fraction of water vapor is less than 10%, which is usually the case when it results from the combination of ambient humidity and products of combustion, then for performance calculations the gas mixture may still be considered perfect.

A physical description of a perfect gas is that its enthalpy is only a function of temperature and not pressure, as there are no intermolecular forces to absorb or release energy when density changes.

Thermodynamic potentials (functions)

u - Internal Energy, $\frac{\mathrm{J}}{\mathrm{kg}}$.

In thermodynamics, the internal energy of a system is the energy contained within the system, excluding the kinetic energy of motion of the system as a whole and the potential energy of the system as a whole due to external force fields. It keeps account of the gains and losses of energy of the system that are due to changes in its internal state.

The internal energy of a system can be changed by transfers of matter or heat or by doing work. When matter transfer is prevented by impermeable containing walls, the system is said to be closed. Then the first law of thermodynamics states that the increase in internal energy is equal to the total heat added plus the work done on the system by its surroundings. If the containing walls pass neither matter nor energy, the system is said to be isolated and its internal energy cannot change. The first law of thermodynamics may be regarded as establishing the existence of the internal energy.

In case of an ideal gas, we can derive that

$$\frac{du}{dT} = c_v .$$

i.e. the internal energy of an ideal gas can be written as a function that depends only on the temperature

$$u(T) = \int_{T_0}^{T} c_v(T)\, dT .$$

If we assume that the specific heat at constant volume does not depend on temperature $(c_v = const)$, and assume that $T_0 = 0$, then the value of the internal energy can be approximately estimated by the formula

$$u = c_v \cdot T .$$

$i = u + p \cdot \nu$ -Enthalpy, $\frac{\mathrm{J}}{\mathrm{kg}}$.

Enthalpy is a measurement of energy in a thermodynamic system. It is the thermodynamic quantity equivalent to the total heat content of a system. It is equal to the internal energy of the system plus the product of pressure and volume.

For ideal gas

$$i(T) = \int_{T_0}^{T} c_p(T)\,dT .$$

If $c_p = const$:

$$i = c_p \cdot T .$$

$f = u - T \cdot s$ - Helmholtz Free Energy, $\frac{\mathrm{J}}{\mathrm{kg}}$.

In thermodynamics, the Helmholtz free energy is a thermodynamic potential that measures the "useful" work obtainable from a closed thermodynamic system at a constant temperature and volume. The negative of the difference in the Helmholtz energy is equal to the maximum amount of work that the system can perform in a thermodynamic process in which volume is held constant. If the volume is not held constant, part of this work will be performed as boundary work. The Helmholtz energy is commonly used for systems held at constant volume. Since in this case no work is performed on the environment, the drop in the Helmholtz energy is equal to the maximum amount of useful work that can be extracted from the system. For a system at constant temperature and volume, the Helmholtz energy is minimized at equilibrium.

$g = u + p \cdot v - T \cdot s$ - Gibbs Energy, $\frac{\mathrm{J}}{\mathrm{kg}}$.

The Gibbs energy is a thermodynamic potential that can be used to calculate the maximum of reversible work that may be performed by a thermodynamic system at a constant temperature and pressure (isothermal, isobaric). The Gibbs free energy (J in SI units) is the maximum amount of non-expansion work that can be extracted from a thermodynamically closed system (one that can exchange heat and work with its surroundings, but not matter); this maximum can be attained only in a completely reversible process. When a system transforms reversibly from an initial state to a final state, the decrease in Gibbs free energy equals the work done by the system to its surroundings, minus the work of the pressure forces.

The Gibbs energy is also the thermodynamic potential that is minimized when a system reaches chemical equilibrium at constant pressure and temperature. Its derivative with respect to the reaction coordinate of the system vanishes at the equilibrium point. As such, a reduction in G is a necessary condition for the spontaneity of processes at constant pressure and temperature.

Typical thermodynamic processes

Isobaric process: $p = const.$

Isothermal process: $T = const.$

Isochoric process: $\nu = const.$

Isentropic process: $s = const.$

Stagnation (total) properties

p^* - total pressure, Pa .

T^* - total or stagnation temperature, K .

ρ^* - stagnation flow density, $\frac{\mathrm{kg}}{\mathrm{m}^3}$.

i^* - total enthalpy, $\frac{\mathrm{J}}{\mathrm{kg}}$.

The difference between total and static pressure at a point is called either the dynamic pressure, dynamic head or velocity head. The term head relates back to hydraulic engineering. The ratio of total to static pressure, as for temperature, is a function of only gamma and Mach number. Most performance calculations are conducted using total pressure, that at engine inlet again resulting from ambient static plus intake ram recovery.

Total enthalpy is the sum of the enthalpy associated with the temperature at each point and the enthalpy associated with the kinetic energy of the fluid mass.

Kinematic parameters

An important characteristic of gases is the speed of pressure-wave propagation or, as otherwise called, the speed of sound. From small-pressure-disturbance theory. From the ideal gas law and isentropic process relations, this reduces to

$$a = \sqrt{k \cdot R \cdot T} .$$

The ratio of fluid velocity V to sound velocity a is an important factor in determining the flow characteristics of a gas. This ratio is called the Mach number M:

$$M = \frac{c}{a} .$$

Mach number is a useful parameter not only for identifying flow-behavior regimes, but also for simplifying and generalizing certain expressions.

Another velocity ratio often used is the ratio of fluid velocity to critical velocity

$$\lambda = \frac{c}{a_{\mathrm{cr}}},$$

where a_{cr} is speed of sound at critical condition, in $\frac{\mathrm{m}}{\mathrm{s}}$. The critical velocity is equal to the velocity of sound at the critical condition. The critical condition is that condition where M= 1.

The ratio of fluid velocity to critical velocity is sometimes called the **critical velocity ratio**. Its use is often preferred over Mach number because the critical velocity ratio is directly proportional to velocity, while Mach number is not (since there is a square root of static temperature in the denominator).

Dynamic parameter

Φ - Total Momentum, N .

The total momentum of the flow is determined by the expression:

$$\Phi = G \cdot c + p \cdot F \ .$$

The first term on the right side of this expression is the dynamic component of the total momentum (dynamic momentum), and the second term is static component.

Isentropic Flow relationships

$$\pi(\lambda)=\frac{p}{p^*}=\left(1-\frac{k-1}{k+1}\lambda^2\right)^{\frac{k}{k-1}};$$

$$\tau(\lambda)=\frac{T}{T^*}=\left(1-\frac{k-1}{k+1}\lambda^2\right);$$

$$\varepsilon(\lambda)=\frac{\rho}{\rho^*}=\left(1-\frac{k-1}{k+1}\lambda^2\right)^{\frac{1}{k-1}};$$

$$q(\lambda)=\frac{\rho\cdot c}{\hat{\rho}\cdot\hat{a}}=\frac{\lambda\cdot\varepsilon(\lambda)}{\varepsilon(1)}=\left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}\lambda\left(1-\frac{k-1}{k+1}\lambda^2\right)^{\frac{1}{k-1}}$$ - relative current density;

$$z(\lambda)=\frac{\Phi}{G\cdot V_{\text{cr}}}=\lambda+\frac{1}{\lambda};$$

$$f(\lambda)=\frac{\Phi}{10^3\cdot p^*\cdot F}=\lambda\cdot\varepsilon(\lambda)\cdot z(\lambda)=\left(1+\lambda^2\right)\left(1-\frac{k-1}{k+1}\lambda^2\right)^{\frac{1}{k-1}}.$$

First Law of Thermodynamics (The Energy Equation)

This equation is the mathematical formulation of the law of conservation of energy. It states that the rate at which energy enters the volume of a moving fluid is equal to the rate at which work is done on the surroundings by the fluid within the volume and the rate at which energy' increases within the moving fluid. The energy in a moving fluid is composed of internal, flow, kinetic, and potential energy. For isentropic flow, the energy equation can be written as follows, noting that the addition of internal and flow energies can be written as the enthalpy of the fluid:

$$\left(i_{\mathrm{X}}+\frac{c_{\mathrm{X}}^2}{2}\right)-\left(i_{\mathrm{O}}+\frac{c_{\mathrm{O}}^2}{2}\right)=i_{\mathrm{X}}^*-i_{\mathrm{O}}^*=\pm L\pm Q_{\mathrm{ext}}\,.$$

$$\Delta i^*=\pm L\pm Q_{\mathrm{ext}}\,.$$

Second Law of Thermodynamics

Changes in flow properties must also obey the second law of thermodynamics. Thus,

$$ds = \frac{\delta\left(Q_{\text{int}} + Q_{\text{ext}}\right)}{T} .$$

Relation of State Conditions for Constant Entropy Process

$$\left(\frac{T_{\mathrm{X}}}{T_{\mathrm{O}}}\right)=\left(\frac{p_{\mathrm{X}}}{p_{\mathrm{O}}}\right)^{\left(\frac{k-1}{k}\right)}.$$

Actual conditions within and across the turbine are usually determined from isentropic process calculations in conjunction with some efficiency or loss term. It is, therefore, necessary to be able to relate state conditions for an isentropic process.

The Continuity Equation

The continuity equation is a mathematical formulation of the law of conservation of mass of a gas that is a continuum. The law of mass conservation states that the mass of a volume moving with the fluid remains unchanged

$$G = c \cdot \rho \cdot F .$$

In practice, it is convenient to use the formula for mass flow rate expressed by total parameters (p^* и T^*) and the relative flux density $q(\lambda)$:

$$G = \frac{m \cdot p^* \cdot F \cdot q(\lambda)}{\sqrt{T^*}},$$

where $m = \sqrt{\frac{k}{R}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}$.

The following values can be used for the m coefficient at approximate calculations:

$m_a = 0.0404\sqrt{\frac{\mathrm{kg}\cdot\mathrm{K}}{\mathrm{J}}}$ - for air;

$m_{cg} = 0.0397\sqrt{\frac{\mathrm{kg}\cdot\mathrm{K}}{\mathrm{J}}}$ - for combustion products of aviation kerosene.

The Momentum Equation—Newton's Second Law of Motion

The momentum equation relates the sum of the external forces acting on a fluid element to the rate of change of momentum in the direction of the resultant external force.

$$\Phi_X - \Phi_O = P_{ext} .$$

In the turbomachinery many applications of the momentum equation can be found, e.g. the force exerted upon a blade in a compressor or turbine cascade caused by the deflection or acceleration of fluid passing the blades.